Définitions

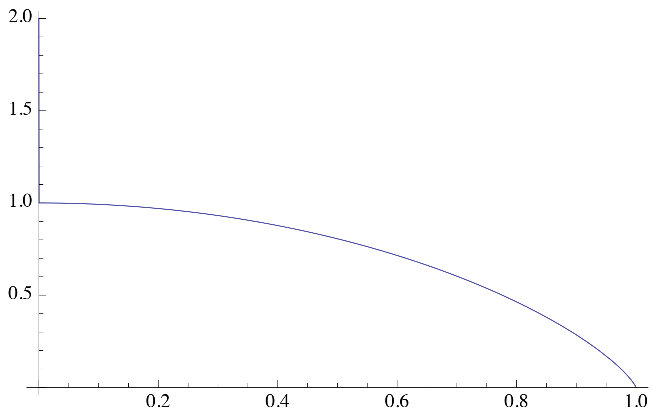
Fonction de Hertz:

```
H[e_] = ((1 - e^2) (EllipticE[e^2] - EllipticK[e^2])) / ((1 - e^2) EllipticK[e^2] - EllipticE[e^2]);
H[0] = 1;
K[e_] = EllipticK[e^2];
Ell[e_] = EllipticE[e^2];
```

```
Limit[H[e], e → 0]
```

1

```
Plot[H[e], {e, 0, 1}]
```



Exemples numériques (extraits de Foulon)

Problème 1

Rayons de courbure:

```
N[R1] = -4 * 10^-3;
N[R1p] = 325 * 10^-5;
N[R2] = -4 * 10^-3;
N[R2p] = 325 * 10^-5;
```

Courbures:

```
ρ1 = 1 / R1;
ρ1p = 1 / R1p;
ρ2 = 1 / R2;
ρ2p = 1 / R2p;
```

Angle entre les directions de directions de courbures minimales:

```
N[θ] = π / 2;
```

Modules d'élasticité longitudinale:

```
N[E1] = 210 * 10^9;
N[E2] = 210 * 10^9;
```

Coefficients de Poisson:

$N[\nu_1] = 3 / 10;$
$N[\nu_2] = 3 / 10;$

Pression à l'équilibre statique:

```
N[F] = 1000;
```

Définition des constantes A et B:

$$A = \frac{1}{4}\left(\rho_1 + \rho_2 + \rho_{1p} + \rho_{2p} - \sqrt{\left(\left(\rho_1 - \rho_{1p}\right)^2 + 2\,\mathrm{Cos}[2\,\theta]\,\left(\rho_2 - \rho_{2p}\right)\left(\rho_1 - \rho_{1p}\right) + \left(\rho_2 - \rho_{2p}\right)^2\right)}\right);$$

$$B = \frac{1}{4}\left(\rho_1 + \rho_2 + \rho_{1p} + \rho_{2p} + \sqrt{\left(\left(\rho_1 - \rho_{1p}\right)^2 + 2\,\mathrm{Cos}[2\,\theta]\,\left(\rho_2 - \rho_{2p}\right)\left(\rho_1 - \rho_{1p}\right) + \left(\rho_2 - \rho_{2p}\right)^2\right)}\right);$$

```
{A, B} // N
```

{28.84615384615384, 28.84615384615384}

Définition des ϵ_i:

$$\varepsilon_1 = \frac{1 - \nu_1^2}{\pi\,E_1};$$

$$\varepsilon_2 = \frac{1 - \nu_2^2}{\pi\,E_2};$$

$\{\varepsilon_1, \varepsilon_2\}$ // N

$\{1.37934 \times 10^{-12}, 1.37934 \times 10^{-12}\}$

```
Clear[e];
e = e /. FindRoot[H[e] == N[A / B], {e, 0.3}, PrecisionGoal → 15]
```

0.000155079

Comme A = B, on a ici e = 0;

Vérification:

```
H[0]
```

1

$$d = (\varepsilon_1 + \varepsilon_2) * \frac{3\,F}{2}$$

$$\frac{3}{2}\,F\left(\frac{1 - \nu_1^2}{\pi\,e_1} + \frac{1 - \nu_2^2}{\pi\,e_2}\right)$$

```
d // N
```

4.13803×10^{-9}

Calcul de a, b et δ:

$$a = \sqrt[3]{\frac{d\,(K[e] - Ell[e])}{e^2\,A}} \quad // \; N$$

0.000482983

$$b = \sqrt[3]{\frac{d\sqrt{1-e^2}\,\left(\text{Ell}[e] - \left(1-e^2\right)\text{K}[e]\right)}{e^2\,B}}\ \text{// N}$$

0.000482983

$$\delta = \sqrt[3]{\frac{d^2\,e^2\,A\,\text{K}[e]^3}{\text{K}[e] - \text{Ell}[e]}}\ \text{// N}$$

0.000013458

Définition de la pression:

$$\text{pression}[x\_, y\_] = \frac{3\,F}{2\,\pi\,a\,b}\sqrt{1 - \frac{x^2}{a^2} - \frac{y^2}{b^2}}\ \text{// N}$$

$2.04681\times 10^9\sqrt{1. - 4.28683\times 10^6\,x^2 - 4.28683\times 10^6\,y^2}$

Pression maximale:

pression[0, 0] // N

2.04681×10^9

$$\alpha = \sqrt[3]{\frac{2\,\text{Ell}[e]}{\pi\left(1-e^2\right)}}$$

1.

$$\beta = \sqrt[3]{\frac{2\sqrt{1-e^2}\,\text{Ell}[e]}{\pi}}$$

1.

$$\gamma = \frac{1}{\alpha\,\beta}$$

1.

$$\lambda = \sqrt[3]{\frac{4\left(1-e^2\right)\text{K}[e]^3}{\pi^2\,\text{Ell}[e]}}$$

1.

Tests:

On retrouve a:

$$\sqrt[3]{\frac{\pi\,d\,\alpha^3}{2\,(A+B)}}\ \text{// N}$$

0.000482983

On retrouve b:

$$\sqrt[3]{\frac{\pi\, d\, \beta^3}{2\,(A+B)}}\ \text{// N}$$

0.000482983

On retrouve δ:

$$\sqrt[3]{\frac{(A+B)\,\pi^2\, d^2\, \lambda^3}{4}}\ \text{// N}$$

0.000013458

On retrouve pmax:

$$\sqrt[3]{\frac{27\,(A+B)^2\, F^3\, \gamma^3}{2\,\pi^5\, d^2}}\ \text{// N}$$

2.04681×10^9

$$\frac{3\,F}{2\,\pi\, a\, b}\ \text{// N}$$

2.04681×10^9

```
pression[0, 0] // N
```

2.04681×10^9

```
b
```

0.000482983

```
b / a
```

1.

$$\frac{\beta}{\alpha}$$

1.

$$\sqrt{1 - e^2}$$

1.

- **Problème 2**

Rayons de courbure:

$N[R_1] = -5 * 10^{-3};$
$N[R_{1p}] = 3.5 * 10^{-3};$
$N[R_2] = -4 * 10^{-3};$
$N[R_{2p}] = 3.25 * 10^{-3};$

Courbures:

$\rho_1 = 1 / R_1;$
$\rho_{1p} = 1 / R_{1p};$
$\rho_2 = 1 / R_2;$
$\rho_{2p} = 1 / R_{2p};$

Angle entre les directions de directions de courbures minimales:

```
N[θ] = π / 2;
```

Modules d'élasticité longitudinale:

```
N[E1] = 210 * 10^9;
N[E2] = 210 * 10^9;
```

Coefficients de Poisson:

```
N[ν1] = 0.3;
N[ν2] = 0.3;
```

Pression à l'équilibre statique:

```
N[P] = 1000;
```

Définition des constantes A et B:

$$A = \frac{1}{4}\left(\rho_1 + \rho_2 + \rho_{1\,p} + \rho_{2\,p} - \sqrt{\left(\left(\rho_1 - \rho_{1\,p}\right)^2 + 2\,\mathrm{Cos}[2\,\theta]\left(\rho_2 - \rho_{2\,p}\right)\left(\rho_1 - \rho_{1\,p}\right) + \left(\rho_2 - \rho_{2\,p}\right)^2\right)}\right);$$

$$B = \frac{1}{4}\left(\rho_1 + \rho_2 + \rho_{1\,p} + \rho_{2\,p} + \sqrt{\left(\left(\rho_1 - \rho_{1\,p}\right)^2 + 2\,\mathrm{Cos}[2\,\theta]\left(\rho_2 - \rho_{2\,p}\right)\left(\rho_1 - \rho_{1\,p}\right) + \left(\rho_2 - \rho_{2\,p}\right)^2\right)}\right);$$

```
{A, B} // N
```

```
{17.8571, 53.8462}
```

Définition des ϵ_i:

$$\varepsilon_1 = \frac{1 - \nu_1^2}{\pi\,E_1};$$

$$\varepsilon_2 = \frac{1 - \nu_2^2}{\pi\,E_2};$$

```
{ε1, ε2} // N
```

$$\left\{1.37934 \times 10^{-12},\ 1.37934 \times 10^{-12}\right\}$$

```
Clear[e];
e = e /. FindRoot[H[e] == N[A / B], {e, 0.3}]
```

```
0.876715
```

Vérification:

```
H[e]
```

```
0.331633
```

```
A / B // N
```

```
0.331633
```

$$d = (\varepsilon_1 + \varepsilon_2) * \frac{3\,F}{2}$$

$$\frac{3}{2}\,F\left(\frac{1 - \nu_1^2}{\pi\,e_1} + \frac{1 - \nu_2^2}{\pi\,e_2}\right)$$

```
d // N
```

$$4.13803 \times 10^{-9}$$

Calcul de a, b et δ:

$$a = \sqrt[3]{\frac{d\,(K[e] - Ell[e])}{e^2\,A}}\ //\ N$$

0.000668769

$$b = \sqrt[3]{\frac{d\,\sqrt{1-e^2}\,\left(Ell[e] - \left(1-e^2\right) K[e]\right)}{e^2\,B}}\ //\ N$$

0.000321685

$$\delta = \sqrt[3]{\frac{d^2\,e^2\,A\,K[e]^3}{K[e] - Ell[e]}}\ //\ N$$

0.0000135587

Définition de la pression:

$$pression[x\_, y\_] = \frac{3\,F}{2\,\pi\,a\,b}\sqrt{1 - \frac{x^2}{a^2} - \frac{y^2}{b^2}}\ //\ N$$

$$2.2194\times 10^9\,\sqrt{1. - 2.23588\times 10^6\,x^2 - 9.6636\times 10^6\,y^2}$$

Pression maximale:

```
pmax = pression[0, 0] // N
```

2.2194×10^9

$$\alpha = \sqrt[3]{\frac{2\,Ell[e]}{\pi\left(1-e^2\right)}}$$

1.48874

$$\beta = \sqrt[3]{\frac{2\sqrt{1-e^2}\,Ell[e]}{\pi}}$$

0.716098

$$\gamma = \frac{1}{\alpha\,\beta}$$

0.938014

$$\lambda = \sqrt[3]{\frac{4\left(1-e^2\right)K[e]^3}{\pi^2\,Ell[e]}}$$

0.93705

Tests:

$$\sqrt[3]{\frac{\pi\, d\, \alpha^3}{2\ (A + B)}}\ \ //\ N$$

0.000668769

a

0.000668769

$$\sqrt[3]{\frac{\pi\, d\, \beta^3}{2\ (A + B)}}\ \ //\ N$$

0.000321685

b

0.000321685

$$\sqrt[3]{\frac{(A + B)\ \pi^2\, d^2\, \lambda^3}{4}}\ \ //\ N$$

0.0000135587

δ

0.0000135587

$$\sqrt[3]{\frac{27\ (A + B)^2\, F^3\, \gamma^3}{2\, \pi^5\, d^2}}\ \ //\ N$$

2.2194×10^9

pmax

2.2194×10^9

$$\frac{3\, F}{2\, \pi\, a\, b}\ //\ N$$

2.2194×10^9

b / a

0.48101

$$\frac{\beta}{\alpha}$$

0.48101

$$\sqrt{1 - e^2}$$

0.48101